ASSIGNMENT 1

1. Solve the partial differential equation

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x$$

2. Solve the partial differential equation

$$(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{2x+y}$$

3. Solve the partial differential equation

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x+2y)$$

4. Solve the partial differential equation

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy$$

5. Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ where } u(x,0) = 6e^{-3x}\ ;\ x > 0,\ t > 0$$

6. Solve the equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given $u = 3e^{-y} - e^{-5y}$ when $x = 0$

7. A thin uniform tightly stretched vibrating string fixed at the points $x = 0$ and $x = l$ satisfies the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$; $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$ and released from rest from this position. Find the displacement $y(x,t)$ at any x and any time t .

8. Solve the differential equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u = \sin t$ at $x = 0$ and $\frac{\partial u}{\partial x} = \sin t$ at $x = 0$.

9. A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to $0^o c$ and are kept at that temperature. Prove that the temperature function $u(x,t)$ is given by

$$u(x,t) = \frac{4u_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\frac{(2n-1)\pi x}{l}e^{-\frac{c^2(2n-1)^2\pi^2 t}{l^2}} .$$

10. Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions

(i) $u \to 0$ as $y \to \infty$ for all x (ii) $u = 0$ at $x = 0$ for all y

(iii) $u = 0$ at $x = l$ for all y (iv) $u = lx - x^2$ if $y = 0$ for all $x \in (0,l)$

ASSIGNMENT 2

1. A machine has 2 major components **A** and **B**. The machine stops functioning if at least one of **A** or **B** goes out of order. Probability **A** goes out of order is **0.4**, probability **B** goes out of order is **0.5**, both **A** and **B** are out of order is **0.15**. What is the probability machine does not function?

2 A bag contains **6** white and **9** black balls. Four balls are drawn at a time. Find the probability for the first draw to give **4** white and the second to give **4** black balls when the balls are not replaced before the second draw.

3. A box **A** contains **2** white and **4** black balls. Another box **B** contains **5** white and **7** black balls. A ball is transferred from the box **A** to the box **B.** Then a ball is drawn from the box **B.** Find the probability that it is white.

4. A problem in mathematics is given to three students **A, B,** and **C** whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

5. In a toy factory, machines ***A, B*** and ***C*** manufacture respectively **25%, 35%** and **40%** of the total. Of their output **5, 4, 2** percents are respectively defective. A toy is drawn at random from the total production. What is the probability that the toy drawn is defective? Also find the probability that it was manufactured by machine ***A***.

6. Determine the value of ***k,*** if the probability function of a random variable ***X*** is given by

$$p(x)=\begin{cases}\frac{kx}{20}, & x=1,2,3,4\\ 0 & \text{for other integers}\end{cases}.$$

7. ***X*** is a continuous random variable with probability density function given by

$$f(x)=\begin{cases}k\,x & ;\ 0\le x<5\\ k(10-x) & ;\ 5\le x<10\\ 0 & ;\ \text{otherwise}\end{cases}.$$

(i) Find the value of ***k*** **(ii)** Mean of ***X*** **(iii)** $P(5<X\le 12)$.

8. The first four moments of a distribution about the value **4** of the variable are **1, 4, 10** and **45** respectively. Find the mean and all the four moments about the mean. Also comment upon skewness and kurtosis.

9. A random variable ***X*** has probability function $p(x)=\frac{1}{2^x}$, $x=1,2,3,\cdots$. Find its moment generating function about origin.

10. Find the moment generating function of the distribution

$$f(x)=\frac{1}{c}e^{-x/c},\ 0\le x<\infty,\ c>0$$

about origin. Hence find its mean and standard deviation.

ASSIGNMENT 3

1. The probability that a bomb dropped from a plane will hit the target is **0.2**. If **6** bombs are dropped, find the probability that
(i) exactly two will hit the target
(ii) at least two will hit the target.

2. Show that Poisson distribution is a limiting case of Binomial distribution when n is very large and p is small such that np is fixed. Also, using Poisson distribution, find the chance that out of **2,000** individuals more than two will get a bad reaction, if the probability of a bad reaction from a certain injection is **0.001** .

3. In a normal distribution, **7%** of the items are under **35** and **89%** are under **63**. Find the mean and S.D. of the distribution? Given that

$$P(0 \le Z \le 0.18) = 0.07, \; P(0 \le Z \le 1.48) = 0.43, P(0 \le Z \le 1.23) = 0.39$$

4. By the method of least squares, fit a straight line $y = mx + c$ from the following data:

x :	**1**	**2**	**3**	**4**	**5**
y:	**14**	**13**	**9**	**5**	**2**

5. By the method of least squares, fit a parabola from the following data:

x :	**1**	**2**	**3**	**4**	**5**
y:	**2**	**6**	**4**	**5**	**2**

6. Two random variables have the regression lines with equation $3x + 2y = 26$ and $6x + y = 31$. Find the mean values of x and y. Also, find the correlation coefficient between x and y.

7. If θ is the acute angle between two regression lines, show that

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2},$$ where σ_x and σ_y are the S.D.'s of x and y-series respectively and r is the correlation coefficient.

8. The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the regression coefficients b_{yx} , b_{xy} and the correlation coefficient r. Also, find the standard deviation of y, if the variance of x is **4**.

9. Find the coefficient of correlation for the data:

$$n = 10, \sum x = 20, \sum y = 40, \sum x^2 = 240, \sum y^2 = 410, \sum xy = 200.$$

10. Calculate the rank correlation coefficient from the following data showing ranks of **5** students in two subjects:

Maths :	**3**	**2**	**4**	**1**	**5**
Chemistry :	**5**	**4**	**3**	**2**	**1**

ASSIGNMENT 4

1. A normally distributed population has means **6.8** and standard deviation **1.5**. A sample of size **400** has mean **6.75**. Is the difference between the population mean and the sample mean significant?

2. The means of two large samples of **1000** and **2000** members are **168.75** cm and **170** cm respectively. Can the samples be regarded as drawn from the same population of standard deviation **6.25** cm?

3. A machinist is making engine parts with axle diameter of **0.7** inch. A random sample of **10** parts shows mean diameter **0.742** inch with a standard deviation of **0.04** inch. On the basis of this sample, would you say that the work is inferior?

4. A sample of nine items has the following values: **45, 47, 50, 52, 48, 47, 49, 53, 51**. Does the mean of sample differ significantly from the population mean **47.5**?

5. A tea company claims that its premium tea brand outsells its normal brand by **10%**. If it is found that **46** out of a sample of **200** tea-users prefer premium brand and **19** out of another independent sample of **100** tea users prefer normal brand. Test the validity of the claim made by the company at **5%** level of significance.

6. The theory predicts the proportion of beans in four groups ***A, B, C*** and ***D*** should be **9:3:3:1.** In an experiment among **1600** beans, the numbers in four groups were **882, 313, 287** and **118.** Does the experimental result support the theory? (Given that $\chi^2_{0.05}$ for **3** d.f.=**7.815**)

7. Two samples of sizes **9** and **8** give the sum of squares of deviations from their respective means as **160** and **91** square units respectively. Test whether the samples can be regarded as drawn from two normal populations with the same variance.

8. A survey of **800** families with four children each recorded the following data:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Test the hypothesis that male and female births are equally likely.

9. Two gauge operations are tested for precision in making measurements. One operator completes a set of **26** readings with a standard deviation of **1.34** and the other does **34** readings with a standard deviation of **0.98**. What is the level of significance of this difference? (Given that for $\nu_1 = 25$ and $\nu_2 = 33$, $z_{0.05} = 0.305$, $z_{0.01} = 0.432$)

10. Two random samples from two normal populations are given below:

Sample I :	16	26	27	23	24	22
Sample II:	33	42	35	32	28	31

Do the estimates of population variances differ significantly?

(Given that $F_{0.05}(5,5) = 5.05$, $F_{0.05}(5,6) = 4.39$, $F_{0.05}(6,5) = 4.95$)

ASSIGNMENT 5

1. A company manufacturers two types of cloth, using three different colors of wool. One yard length of type **A** cloth requires 4 oz of red wool, 5 oz of green wool and 3 oz of yellow wool. One yard length of type **B** cloth requires 5 oz of red wool, 2 oz of green wool and 8 oz of yellow wool. The wool available for manufacturer is 1000 oz of red wool, 1000 oz of green wool and 1200 oz of yellow wool. The manufacturer can make a profit of Rs. 5 on one yard of type **A** cloth and Rs. 3 on one yard of type **B** cloth. Formulate this problem as a linear programming problem to find the best combination of the quantities of type **A** and type **B** cloth which gives him maximum profit.

2. Using graphical method, solve the following L.P.P.

$$\begin{aligned} \text{Min } Z &= x_1 + x_2 \\ s.t. \quad x_1 + 2x_2 &\le 10 \\ x_1 + x_2 &\ge 1 \\ x_1 &\le 4 \\ x_1 \, , \, x_2 &\ge 0. \end{aligned}$$

3. Using graphical method, find the maximum value of

$$\begin{aligned} Z &= 2x + 3y \\ s.t. \quad x + y &\le 30 \\ y &\ge 3 \\ x &\ge y \\ 0 \le x &\le 20 \\ 0 \le y &\le 12. \end{aligned}$$

4. Convert the following L.P.P. to the standard form:

$$\begin{aligned} \text{Max } Z &= 2x_1 + 3x_2 + 5x_3 \\ s.t. \qquad 6x_1 - 3x_2 &\le 5 \\ 3x_1 + 2x_2 + 4x_3 &\ge 10 \\ 4x_1 + 3x_3 &\le 2 \\ x_1 \, , \, x_2 &\ge 0 \end{aligned}$$

5. Solve the following L.P.P. by simplex method

$$\begin{aligned} \text{Max } Z &= 5x_1 + 3x_2 \\ s.t. \quad 3x_1 + 5x_2 &\le 15 \\ 5x_1 + 2x_2 &\le 10 \\ x_1 \, , \, x_2 &\ge 0 \end{aligned}$$

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6. Solve the following L.P.P. by simplex method

$$\begin{aligned} \textbf{Min } Z &= x_1 - 3x_2 + 3x_3 \\ s.t. \quad 3x_1 - x_2 + 2x_3 &\le 7 \\ 2x_1 + 4x_2 &\ge -12 \\ -4x_1 + 3x_2 + 8x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

7. Write the dual of the following problem:

$$\begin{aligned} \textbf{Max } Z &= 4x_1 + 9x_2 + 2x_3 \\ s.t. \quad 2x_1 + 3x_2 + 2x_3 &\le 7 \\ 3x_1 - 2x_2 + 4x_3 &= 5 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

8. Using dual simplex method solve the following L.P.P.

$$\begin{aligned} \textbf{Max } Z &= -3x_1 - 2x_2 \\ s.t. \quad x_1 + x_2 &\ge 1 \\ x_1 + x_2 &\le 7 \\ x_1 + 2x_2 &\ge 10 \\ x_2 &\ge 3 \\ x_1, x_2 &\ge 0 \end{aligned}$$

9. An engineer wants to assign 3 jobs J_1, J_2, J_3 to three machines M_1, M_2, M_3 in such a way that each job is assigned to some machine and no machine works on more than one job. Find the optimal solution using Hungarian method if the cost matrix is as follows:

	M_1	M_2	M_3
J_1	15	10	9
J_2	9	15	10
J_3	10	12	8

10. Solve the following transportation problem by VAM method

Destination

	From\To	A	B	C	D	Availability
Source	I	21	16	25	13	11
	II	17	18	14	23	13
	III	33	27	18	41	19
	Requirement	6	10	12	15	43

ANSWERS (ASSIGNMENT 1)

Here ϕ_1,ϕ_2,ϕ_3 are arbitrary functions.

1. $z=\phi_1(y-x)+\phi_2(y+2x)+ye^x$

2. $z=\phi_1(y-2x)+\phi_2(y-x)+\phi_3(y+3x)-\frac{1}{75}\cos(x+2y)-\frac{1}{12}e^{2x+y}$

3. $z=\phi_1(y-x)+e^{2x}\phi_2(y-x)+\frac{1}{39}[2\cos(x+2y)-3\sin(x+2y)]$

4. $z=e^{-2x}\phi_1(y+2x)+\phi_2(y-x)-\frac{1}{10}e^{2x+3y}-\frac{1}{24}(2x^3-6x^2y-9x^2+6xy+12x)$

5. $u(x,t)=6e^{-(3x+2t)}$ 6. $u(x,y)=3e^{x-y}-e^{2x-5y}$

7. $y(x,t)=\frac{y_0}{4}\left[3\cos\left(\frac{\pi ct}{l}\right)\sin\left(\frac{\pi x}{l}\right)-\cos\left(\frac{3\pi ct}{l}\right)\sin\left(\frac{3\pi x}{l}\right)\right]$ 8. $u(x,t)=\sin t\left(\cos\frac{x}{2}+2\sin\frac{x}{2}\right)$

10. $u(x,y)=\frac{8l^2}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}e^{-\frac{(2n-1)\pi y}{l}}\sin\frac{(2n-1)\pi x}{l}$

ANSWERS (ASSIGNMENT 2)

1. 0.75 2. $\frac{3}{715}$ 3. $\frac{16}{39}$ 4. $\frac{3}{4}$ 5. $\frac{69}{2000}$, $\frac{25}{69}$ 6. $k=2$ 7. (i) $k=\frac{1}{25}$, (ii) 5, (iii) $\frac{1}{2}$

8. $\text{Mean}=5$, $\mu_1=0$, $\mu_2=3$, $\mu_3=0$, $\mu_4=26$, Distribution is symmetrical and platykurtic.

9. $\frac{e^t}{(2-e^t)}$ 10. $M_0(t)=\frac{1}{(1-ct)}$, $\text{Mean}=\text{S.D.}=c$

ANSWERS (ASSIGNMENT 3)

1. (i) 0.246 (ii) 0.345 2. 0.32 3. $\text{Mean}=50.3$, $\text{S.D.}=10.34$ 4. $y=18.2-3.2x$

5. $y=-1.4+4.61x-0.79x^2$ 6. $\bar{x}=4$, $\bar{y}=7$, $r=-0.5$ 8. $b_{yx}=b_{xy}=-\frac{3}{4}$, $r=-\frac{3}{4}$, $\sigma_y=2$

9. $r=0.536$ 10. -0.3

ANSWERS (ASSIGNMENT 4)

1. No 2. No 3. Yes 4. No 5. Yes, claim is valid at 5% level of significance.

6. Yes 7. Yes 8. The hypothesis that male and female births are equally likely is rejected.

9. The difference between the standard deviation is significant at 5% level and insignificant at 1% level of significance. 10. No

ANSWERS (ASSIGNMENT 5)

1. Suppose the manufacturer decide to produce x_1 yards of type **A** cloth and x_2 yards of type **B** cloth. Then

$$\begin{aligned} &\text{Max } Z = 5x_1 + 3x_2 \\ s.t. \quad &4x_1 + 5x_2 \le 1000 \\ &5x_1 + 2x_2 \le 1000 \\ &3x_1 + 8x_2 \le 1200 \\ &x_1, x_2 \ge 0 \end{aligned}$$

2. $\text{Min } Z = 1$ on every point on the line $x_1 + x_2 = 1$ in the first quadrant.

3. $x = 18$, $y = 12$ **and** $\text{Max } Z = 72$

4.

$$\begin{aligned} &\text{Max } Z = 2x_1 + 3x_2 + 5x_3' - 5x_3'' \\ s.t. \quad &6x_1 - 3x_2 + s_1 = 5 \\ &3x_1 + 2x_2 + 4x_3' - 4x_3'' - s_2 = 10 \\ &4x_1 + 3x_3' - 3x_3'' + s_3 = 2 \\ &x_1 \,,\, x_2 \,,\, x_3' \,,\, x_3'' ,\, s_1 \,,\, s_2 \,,\, s_3 \ge 0 \end{aligned}$$

5. $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$ **and** $\text{Max } Z = \frac{235}{19}$

6. $x_1 = \frac{31}{5}$, $x_2 = \frac{58}{5}$ **and** $x_3 = 0$, $\text{Min } Z = \frac{-143}{5}$

7.

$$\begin{aligned} &\text{Min } Z_d = 7y_1 + 5y_2 \\ s.t. \quad &2y_1 + 3y_2 \le 4 \\ &3y_1 - 2y_2 \le 9 \\ &2y_1 + 4y_2 \le 2 \\ &y_1 \ge 0 \,,\, y_2 \text{ is unrestricted in sign} \cdot \end{aligned}$$

8. $x_1 = 4$, $x_2 = 3$ **and** $\text{Max } Z = -18$

9. $J_1 \to M_2$, $J_2 \to M_1$, $J_3 \to M_3$, $\text{Min cost} = 27$

10. From source **I** transport **11** units to destination ***D***;

From source **II** transport **6, 3, 4** units to destinations ***A, B, D*** respectively;

From source **III** transport **7, 12** units to destinations ***B, C*** respectively;

Optimal transport cost = Rs. **796**