Assignment-I Maths III

1. Find the Fourier series expansion of the function $f(x) = \frac{(\pi - x)^2}{4}$ in $(0, 2\pi)$.
2. If $f(x + 2\pi) = f(x)$, find the Fourier series expansion of the function

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ \frac{1}{2}, & x = 0 \\ x, & 0 < x < 1 \end{cases}$$

3. Assuming $f(x) = x$ to be periodic with a period 2π, find Fourier series expansion of $f(x)$ in the interval $(-\pi, \pi)$.
4. Find the Fourier sine series expansion of the function

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$$

5. The following values of 'y' give the displacement of a machine part for the rotation x of a flywheel. Express 'y' in Fourier series.

x	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0
y	70	886	1293	1400	1307	814	-70	-886	-1293	-1400	-1307	-814

Express y in Fourier series up to third harmonics.

6. Solve the integral equation $\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$, $\lambda > 0$
7. Obtain Fourier sine integral of the function $f(x) = \begin{cases} x\,, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0\,, & x > 2 \end{cases}$
8. Prove that Fourier integral of the function $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & otherwise \end{cases}$ is given by $f(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$.
9. Find the Fourier sine and cosine transforms of xe^{-ax}
10. The temperature $u(x, t)$ at any point of a semi infinite bar satisfies the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$, subject to conditions

    i. $u(0, t) = 0, t > 0$

    ii. . $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

    Determine the expression for $u(x, t)$

11. Form a difference equation satisfied by the relation $y = An + B2^n$
12. Solve the Fibonacci difference equation $y_n = y_{n-1} + y_{n-2}, \quad n > 2$
13. Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 4^n$
14. Solve the difference equation $y_{x+2} - 2y_{x+1} + y_x = 3x + 4$
15. Solve the difference equation $y_{n+2} - 6y_{n+1} + 8y_n = 2^n + 6n$
16. Find the Z–transform of $u_n = \begin{cases} 2^n, & n < 0 \\ 3^n, & n \geq 0 \end{cases}$
17. Find the Z–transform of $u_n = \frac{1}{(n-p)!}$
18. Find the inverse Z–transform of $u_n = \frac{2z}{(z-1)(z^2+1)}$
19. Solve the difference equation $y_{x+2} + 4y_{x+1} + 3y_x = 3^x$, $y_0 = 0, y_1 = 1$ using Z–transforms
20. State and prove convolution theorem for Z –transforms