Assignment-II Mathematics III

1. Find the real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places using Bisection method.
2. Perform three iterations to find root of the equation $xe^x - \cos x = 0$ using Regula-Falsi method.
3. Find the real root of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places using Newton- Raphson method.
4. Solve the system of equations:

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$2x_1 - 10x_2 + x_3 + x_4 = -15$$
$$x_1 + x_2 - 10x_3 + 2x_4 = -27$$
$$x_1 + x_2 + 2x_3 - 10x_4 = 9$$

   using Gauss- Jacobi method. Compute results for 2 iterations.
5. Solve the system of equations given in Q4 upto 2 iterations, using Gauss-Seidal method.
6. If $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 2000, u_4 = 100$, calculate $\Delta^4 u_0$.
7. Find the missing value in the following table

x	0	5	10	15	20	25
y	6	10	-	17	-	31

8. Find approximate value of $\cos 23^0$ using interpolation from the given data

x	10^0	20^0	30^0	40^0	50^0	60^0	70^0	80^0
$\cos x$	.9848	.9397	.8660	.7660	.6428	.5	.3420	.1737

9. Find the 4^{th} order divided differences from the given data

x	0.5	1.5	3.0	5.0	6.5	8.0
y	1.625	5.875	31	131	282.125	521

10. If $f(1) = 1, f(3) = 27, f(4) = 64,$ obtain a unique polynomial $f(x)$ of degree less than or equal to 2, using Lagrange's interpolation formula.
11. For the following data, approximate $\frac{dy}{dx}$, at $x = 2$

x	0	2	3
y	2	-2	-1

12. From the data given below, find the value of x , for which y is maximum.

x	3	4	5	6	7	8
y	0.205	0.24	0.259	0.262	0.25	0.224

Also find maximum y.

13. Derive Newton-Cote's quadrature formula for evaluating $\int_a^b f(x)\,dx$, where the interval $[a, b]$ is divided into n subintervals of height h.

14. Approximate $\int_0^6 \frac{1}{1+x^2}\,dx$ using

i. Trapezoidal rule ii. Simpsons $1/3$ rule iii. Simpsons $3/8$ rule and compare the results with direct integration.

15. The velocity v of an airplane, which starts from rest, is given at fixed intervals of time as shown below:

$v(km)$	2	4	6	8	10	12	14	16	18	20
$t(minutes)$	10	18	25	29	32	20	11	5	2	0

Find the approximate distance covered in 20 minutes.

16. If $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y\,(0) = 1$, find the value of y for $x = 0.1$ using Picard's method.

17. Use Taylor's series method to find approximate value of y at $x = 0.2$, given that $\frac{dy}{dx} = 2y + 3e^x$, $y\,(0) = 0$. Compare the results by solving the differential equation directly.

18. Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y\,(0) = 1$, find the value of y for $x = 0.1$ using Euler's method.

19. If $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, $y\,(0) = 1$, find the value of y for the range $0 \le x \le 0.6$ in steps of 0.2 using modified Euler's method.

20. Given that $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y\,(0) = 1$, find the value of y for $x = 0.2$ using Runge-Kutta method of 4^{th} order.